Divisional Semantics for Aristotle's Apodeictic Syllogistic (Outline)

Hunan Rostomyan / March 17, 2013

Abstract An attempt is outlined to (i) extend Vlasits' divisional semantics for Aristotle's assertoric syllogistic[1] to cover the apodeictic fragment of Aristotle's modal syllogistic by situating divisional models in a possible-worlds setting (§1), and to (ii) prove the soundness (§2) and completeness (§3) of apodeictic syllogistic with respect to McCall's axiomatization.[2]

1.1 The Syntax of Apodeictic Syllogistic

Definition 1.1 (Language $\mathcal{L}_\mathrm{N}$) *Given a set of terms* $\mathcal{T} = \{a, b, c, ...\}$*, and a vocabulary* $V_\mathrm{N} = \mathcal{T} \cup \{\neg, \rightarrow, \Box, \mathrm{A}, \mathrm{I}\}$ *we define the language of apodeictic syllogistic* $\mathcal{L}_N$ *as* $(\forall x, y \in \mathcal{T})$:

$$\phi \ ::= \ x\mathrm{A}y \ \mid \ x\mathrm{I}y \ \mid \ \neg\phi \ \mid \ (\phi \rightarrow \phi) \ \mid \ \Box\phi$$

Where the intended interpretation of $(x\mathrm{A}y)$ is that (x holds of y), of $(x\mathrm{I}y)$ is that (for some z in $\mathcal{T}$, z holds of both x and y), and of $(\Box\phi)$ is that (ϕ holds of necessity). On the basis of these primitives we define a number of abbreviations:

$$\begin{aligned} x\mathrm{E}y &=_{Df} \neg(x\mathrm{I}y), \\ x\mathrm{O}y &=_{Df} \neg(x\mathrm{A}y), \\ \phi \wedge \phi' &=_{Df} \neg(\phi \rightarrow \neg\phi'), \\ \Diamond\phi &=_{Df} \neg\Box\neg\phi, \end{aligned}$$

with the following intended interpretations: $(x\mathrm{E}y)$ means that (there is no z in $\mathcal{T}$ that holds both of x and y), $(x\mathrm{O}y)$ means that (x doesn't hold of y), $(\phi \wedge \phi')$ means that (it's not the case that if ϕ holds then the negation of ϕ' holds), and $(\Diamond\phi)$ means that (it's not the case that the negation of ϕ holds of necessity).

1.2 Extended Divisional Semantics for Apodeictic Syllogistic

Definition 1.2 (Semantics of $\mathcal{L}_\mathrm{N}$) *We define our apodeictic syllogistic models as triples* $\mathcal{M} = \langle W, D, \{\uparrow_w\}_{w \in W}\rangle$ *with* W *a set of possible worlds,* D *a set of individuals, a set of preorders* $\{\uparrow_w\}_{w \in W}$*, each of which meets the following 'Nxn' condition:*

$$\forall x, y, z \in D\Big(\Big[\ \langle x, y\rangle \in \bigcap_{i \in W} \uparrow_i \quad \& \quad \langle y, z\rangle \in \bigcup_{i \in W} \uparrow_i\ \Big] \implies \langle y, z\rangle \in \bigcap_{i \in W} \uparrow_i\Big)$$

[1] Vlasits, Justin (2012 [Draft]) "Dvisional Semantics for Aristotle's Assertoric Syllogistic," §2.
[2] McCall, Stors (1963) *Aristotle's Modal Syllogisms*, §§14-19.

The intuition behind this beast is the following. The first conjunct of the antecedent says that there's an arrow from X to Y in all possible worlds; the second conjunct says that there's an arrow from Y to Z in some possible world; the consequent says that there's an arrow from Y to Z in all possible worlds. If the antecedent is met, then all arrow types from X go to Y, and some particular arrow, say $\uparrow_u$, goes from Y to Z. The property forces that in these situations all arrow types go from Y to Z.

We define truth in an extended divisional model, at a point, as follows:

$$\begin{array}{lll}
\langle \mathcal{M}, w\rangle \models x\mathrm{A}y & \text{iff} & \langle Y, X\rangle \in \uparrow_w \\
\langle \mathcal{M}, w\rangle \models x\mathrm{I}y & \text{iff} & \exists z \in \mathcal{T} : \langle \mathcal{M}, w\rangle \models xAz \quad \text{and} \quad \langle \mathcal{M}, w\rangle \models yAz \\
\langle \mathcal{M}, w\rangle \models \neg\phi & \text{iff} & \langle \mathcal{M}, w\rangle \not\models \phi \\
\langle \mathcal{M}, w\rangle \models \phi \to \psi & \text{iff} & \langle \mathcal{M}, w\rangle \models \neg\phi \quad \text{or} \quad \langle \mathcal{M}, w\rangle \models \psi \\
\langle \mathcal{M}, w\rangle \models \Box\phi & \text{iff} & \forall v \in W : \langle \mathcal{M}, v\rangle \models \phi
\end{array}$$

1.3 McCall's Axiomatization of Apodeictic Syllogistic

Definition 1.3 (Proof Theory of $\mathcal{L}_{\mathrm{N}}$) *As our proof theory* **N** *of apodeictic syllogistic we'll take McCall's axiomatization of Aristotle's apodeictic syllogistic (the so-called 'L-X-M calculus'), which consists of the following inference rules and axioms:*

Rules

▷ the rule of double negation (RN)

▷ the rule of *modus ponens* (MP)

Axioms

▷ all substitution instances of all propositional tautologies

▷ a strengthened Łukasiewicz axiomatization of assertoric syllogistic[3]

1. xAx
2. $\Box(xIx)$
3. $(yAz \wedge xAy) \to xAz$[4] (Barbara XXX)
4. $(yAc \wedge yIx) \to xIz$ (Datisi XXX)

▷ four NXN moods

[3]What we have here as (2) appears in Łukasiewicz, Jan (1951) *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd Enlarged 1998 Edition, p. 46, in the following un-modalized form: '2. [x] belongs to some [x]'.

[4]We wish to remain agnostic with respect to the question of what's the better way of understanding Aristotle's syllogisms: as *conditionals* (Łukasiewciz), or as *inferences* (Austin, Corcoran, Smith). Whichever option is taken, the axioms, of course will have to appear as conditionals so that we may use modus ponens in deriving various conclusions.

5. $(\Box(yAz) \wedge xAy)) \rightarrow \Box(xAz)$ (Barbara NXN)
6. $(\Box(zEy) \wedge xAy)) \rightarrow \Box(xEz)$ (Cesare NXN)
7. $(\Box(yAz) \wedge xIy)) \rightarrow \Box(xIz)$ (Darii NXN)
8. $(\Box(yEz) \wedge xIy) \rightarrow \Box(xOz)$ (Ferio NXN)

▷ two NNN moods

9. $(\Box(zAy) \wedge \Box(xOy)) \rightarrow \Box(xOz)$ (Baroco NNN)
10. $(\Box(yOz) \wedge \Box(yAx))) \rightarrow \Box(xOz)$ (Bocardo NNN)

▷ the law of apodeictic I-conversion

11. $\Box(xIy) \rightarrow \Box(yIx)$

▷ three laws of modal subordination

12. $\Box(xAy) \rightarrow xAy$
13. $\Box(xIy) \rightarrow xIy$
14. $\Box(xOy) \rightarrow xOy$

On the basis of these axioms, using these laws, McCall goes on to (i) complete Łukasiewicz's axiomatization of assertoric syllogistic (thesis 15, McCall: p. 39); to deduce (ii) 9 laws of *modal subalternation* (theses 16-24, Ibid.: §15), (iii) 8 laws of *modal conversion* (theses 25-32, Ibid.: §16), (iv) 14 laws of *modal subordination* (theses 33-46, Ibid.: §17), etc., deriving all apodeictic theses held to be valid by Aristotle.

We say that $\Gamma \vdash \phi$ *iff* ϕ *is either (i) an axiom of* **N**, *or is (ii) derivable from the axioms of* **N** by means of its rules of inference.

2. **N** is sound with respect to extended divisional models (Outline)

First we show that the axioms of **N** are valid in extended divisional models. Then we show that the rules (MP & RN) preserve validity.

i. It's very easy to show that the axioms are valid. (Axioms 1-2) follow from the fact that arrows are *reflexive*. (Axioms 3-4) are axioms of assertoric syllogistic, so they follow from the facts that (i) assertoric syllogistic is sound with respect to Vlasits' divisional models[5], and that (ii) our extended models extend the ordinary divisional models. (Axiom 5, Barbara NXN) is validated due to the Nxn condition we imposed on the family of arrows (cf. §1.2). The other perfect modal moods (Axioms 6-10), and laws (Axioms 11-14) follow from the same Nxn condition and the fact that arrows are transitive.

ii. It's also not difficult to show that the two rules preserve validity.

3. **N** is (possibly) complete with respect to extended divisional models (Forthcoming)

[5] cf. Vlasits, op. cit., §3